

Mass Shift and Width Broadening of J/ψ in hot gluonic plasma from QCD Sum Rules

Kenji Morita^{1,*} and Su Houn Lee^{1,†}

¹*Institute of Physics and Applied Physics, Yonsei University, Seoul 120-749, Korea*

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We investigate possible mass shift and width broadening of J/ψ in hot gluonic matter using QCD sum rule. Input values of gluon condensates at finite temperature are extracted from lattice QCD data for the energy density and pressure. Although stability of the moment ratio is achieved only up to $T/T_c \simeq 1.05$, the gluon condensates cause a decrease of the moment ratio, which results in change of spectral properties. Using the Breit-Wigner form for the phenomenological side, we find that mass shift of J/ψ just above T_c can reach maximally 200 MeV and width can broaden to dozens of MeV.

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Heavy quarkonia have been regarded as a very useful probe of quark-gluon plasma (QGP) which can be created in relativistic heavy ion collisions. Since Matsui and Satz argued [1] that the J/ψ suppression could be a signature of QGP formation in heavy ion collisions, extensive works have been performed on the subject both experimentally [2] and theoretically [3]. However, in contrast to early expectations, recent lattice calculations suggest J/ψ can survive up to at least $T \sim 1.5T_c$ [4, 5, 6]. Hence, change of spectral properties, which cannot be seen within current resolution of lattice calculations, may exist in QGP and can reflect the properties of the strongly coupled QGP, at temperature not so higher than T_c . One should note, however, that in such a temperature region, the system is highly non-perturbative one [7], and any analyses should consistently treat the non-perturbative aspects of QCD. In this respect, QCD sum rule is a suitable approach that can be used for analyses of hadron properties at these temperatures.

In the present paper, we investigate the behavior of J/ψ in gluonic plasma slightly above T_c using QCD sum rules [8]. The QCD sum rule has been extensively used for studying in-medium properties of both light and heavy hadrons as a reliable and well-establish method. For heavy quark systems, it is more reliable because all relevant condensates are known and their temperature dependence are easily extracted from lattice QCD. Here we follow the method used in analyses of J/ψ in vacuum [9] and in nuclear matter [10, 11] and study J/ψ at finite temperature within the quenched approximation.

We start with the time ordered current-current correlation function

$$(q^\mu q^\nu - q^2 g^{\mu\nu}) \tilde{\Pi}(q) = i \int d^4x e^{iq \cdot x} \langle T[j^\mu(x) j^\nu(0)] \rangle_T \quad (1)$$

where we take $j^\mu = \bar{c} \gamma^\mu c$ for J/ψ . In this work, we set both the medium and $c\bar{c}$ at rest so that $\mathbf{q} = 0$ and $\tilde{\Pi}(q^2)$ becomes the longitudinal part of the polarization tensor. For a large $Q^2 = -q^2 = -\omega^2 \gg 0$, the correlation function can be expressed through OPE as $\tilde{\Pi}(q^2) = \sum_n C_n \langle O_n \rangle_T$ with C_n and O_n being the perturbative Wilson coefficients and operators of mass dimension n , respectively. In heavy quark systems such as J/ψ , the expansion can be entirely expressed by gluonic operators [8, 9, 10, 11]. Here we assume that the relevant

information is contained in the local operators. Note that, in the deconfined phase, this may have to be remedied since non-local contributions coming from the non-vanishing Polyakov loop might become important [12]. The expectation value is taken at finite $T > T_c$ and the temperature dependence can be imposed only to the expectation value of the gluonic operator in the case of $m_c \gg T$ and $T^2 \ll Q^2$ [13]. In previous QCD sum rule works, Hashimoto *et al.* calculated the Wilson coefficient of the scalar gluon operator at finite temperature [14]. In the works of Furnstahl *et al.* [15], scattering contribution was calculated. However, none of these works included the contribution from the gluon operator with spin nor the changes of the gluon operators systematically extracted from the lattice calculations. In contrast, we have included all the lowest non-vanishing operators, have extracted the full temperature dependence of the gluon operators from the quenched lattice data, and have modeled the phenomenological side consistent with the quenched assumption. Hence, this work marks the first systematic application of QCD sum rules slightly above T_c .

Following Ref. [9], let us consider the n -th moment of the correlation function (1),

$$M_n(Q^2) \equiv \frac{1}{n!} \left(\frac{d}{dq^2} \right)^n \tilde{\Pi}(q^2) \Big|_{q^2 = -Q^2}. \quad (2)$$

From the OPE side, up to dimension four, this moment can be expressed as

$$M_n(Q^2) = A_n(\xi) [1 + a_n(\xi) \alpha_s + b_n(\xi) \phi_b + c_n(\xi) \phi_c] \quad (3)$$

where $\xi = Q^2/4m_c^2$ is a dimensionless scale factor, A_n , a_n , b_n and c_n are the Wilson coefficients corresponding to bare loop diagrams, perturbative radiative correction, scalar gluon condensate and twist-2 gluon operator, respectively [10]. Note that there appears the additional twist-2 contribution when we consider the medium expectation value. The Wilson coefficients are listed in Ref. [9, 10]. The explicit forms of ϕ_b and ϕ_c are $\phi_b = \frac{4\pi^2}{9(4m_c^2)^2} G_0$ and $\phi_c = \frac{4\pi^2}{3(4m_c^2)^2} G_2$, where $G_0 = \langle \frac{\alpha_s}{\pi} G_{a\mu\nu} G_a^{\mu\nu} \rangle_T$, and G_2 is the twist-2 condensate contribution defined by $\langle \frac{\alpha_s}{\pi} G_a^{\mu\rho} G_{a\rho}^\nu \rangle_T = (u^\mu u^\nu - \frac{1}{4} g^{\mu\nu}) G_2$ with u^μ being the 4-velocity of the medium. These condensates can be determined from lattice QCD data as follows.

The scalar condensate is related to the energy-momentum tensor through the trace anomaly. If we take 1-loop expression for the beta function of pure SU(3) theory, we get $G_0 = G_0^{\text{vac}} - \frac{8}{11}(\varepsilon - 3p)$ where $G_0^{\text{vac}} = (0.35\text{GeV})^4$ is the value of the gluon condensate in vacuum [16] and the second term comes from the trace anomaly [17]. ε and p are the energy density and pressure, respectively. On the other hand, the twist-2 part can be simply related to the energy-momentum tensor of the pure gauge theory as $T^{\alpha\beta} = -G_a^{\alpha\lambda}G_a^{\beta}_{\lambda}$ ($\alpha \neq \beta$). Hence, recalling that $T^{\alpha\beta} = (\varepsilon + p)u^\alpha u^\beta - pg^{\alpha\beta}$, we obtain $G_2 = -\frac{\alpha_s(T)}{\pi}(\varepsilon + p)$. The thermodynamic quantities and the effective coupling constant $\alpha_s(T)$ are taken from quenched lattice QCD calculation [18, 19]. Accounting for the ambiguities of $\alpha_s(T)$ in the non-perturbative regime, we adopt two of results in Ref. [19] for the temperature dependent coupling constant; one is determined from the short-distant force and the other is from the screening part of the large distant part. The former does not depend on temperature at very short distance and takes its maximum values at some distance r_{screen} which decreases with increasing temperature. We use the value at this distance and denote it as $\alpha_{qq}(T)$ following Ref. [19]. To obtain the temperature dependence, we fit the lattice data point (Fig.6 top in Ref. [19]) by Bezier interpolation, which results in $\alpha_{qq}(T_c) = 0.626$. The latter, which we denote $\tilde{\alpha}(T)$ as in Ref. [19], has a similar value, but error-bars are still too large especially near T_c . We use the two-loop expression of the running coupling constant with a set of parameters given in Ref. [19]. This gives $\tilde{\alpha}(T_c) = 0.47$. The extracted gluon condensates G_0 and G_2 above but near T_c are shown in Fig. 1. One thing to note is that, G_0 decreases to less than half of its vacuum value but remains positive near T_c [20]. We can see that the tensor condensates have non-negligible values near T_c .

The n -th moment in Eq. (2) can also be expressed as

$$M_n(Q^2) = \int_0^\infty \frac{\rho_h(s)}{(s + Q^2)^{n+1}} ds, \quad (4)$$

where $\rho_h(s) = \frac{1}{\pi} \tanh\left(\frac{s}{2T}\right) \text{Im}\Pi(s)$ is the phenomenological spectral function which in general includes not only the pole term but also the continuum and scattering part [15, 21]. The scattering term, which also appears in the OPE side and contributes with a delta function at zero frequency, could be important in the presence of thermal fermion. However, since we are considering the gluonic medium and have extracted the condensates from the pure gauge theory, we can consistently assume that there is no (anti-)quarks which can scatter with the current.

We can put $\tanh\left(\frac{s}{2T}\right) = 1$ due to much larger pole mass and continuum threshold than temperature considered here. Then, hadronic properties such as mass and width are related to the OPE side [Eq. (3)] by putting a phenomenological functional form in $\text{Im}\Pi(s)$ of the above equation. Here we employ a simple Breit-Wigner form

$$\text{Im}\Pi^{\text{pole}}(s) = \frac{f_0 \sqrt{s} \Gamma}{(s - m^2)^2 + s \Gamma^2}, \quad (5)$$

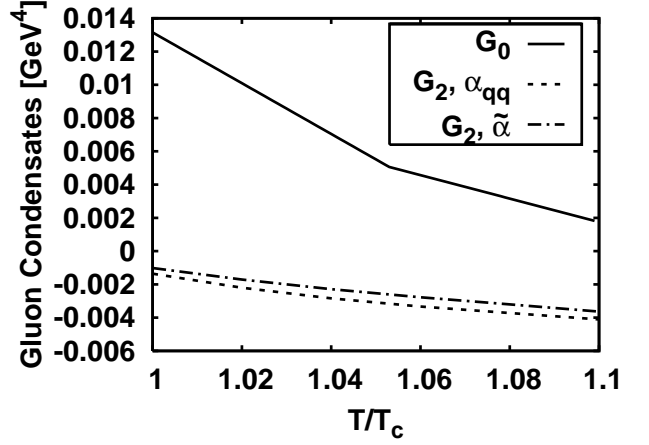


FIG. 1: Gluon condensates as a function of T/T_c .

to take finite width into account. Since we are interested in the lowest lying resonance of the vector channel, we should choose an appropriate order n so that the moment [Eq. (4)] contains information only on the pole term of the spectral function. Following Refs. [8, 9, 10], we take the ratio of the moment $r_n = M_{n-1}/M_n$ and choose moderately large n such that the contribution from the excited states and continuum can be neglected. Therefore, this ratio should approach a constant value at sufficiently large n . However, when n is large, contribution from higher dimension operators becomes important. At the n value where r_n is minimum, pole dominance and truncation of the OPE are valid and the ratio is close to the real asymptotic value, as have been extensively investigated in the vacuum sum rule for J/ψ [9]. In this work we only consider temperature range in which the same criterion can still be applied and take the minimum value for r_n to be its asymptotic value. Hence, in the practical calculation below, we firstly evaluate the appropriate n for various temperatures by calculating $r_n|_{\text{OPE}}$. Then, we look for pairs of m and Γ which satisfy the sum rule relation, $r_n|_{\text{OPE}} = r_n|_{\text{phen.}}$. We employ the vegas monte-carlo integration [22] to treat a very sharp peak in the dispersion integral [Eq. (4)] of the phenomenological side. The relative error in the numerical integration is found to be on the order of 10^{-6} for $m = 3$ GeV and $\Gamma = 1$ MeV. This numerical accuracy becomes better as Γ increases, as naively expected. The normalization scale ξ is chosen as $\xi = 1$. We checked that our result does not strongly depend on the choice of ξ by varying ξ from 0 to 3 [23]. Other parameters of the theory are taken from Ref. [10], $\alpha_s(8m_c^2) = 0.21$ and $m_c = 1.24$ GeV. We did not do fine tuning of these parameters to adjust the vacuum mass of J/ψ since our interest is in the change of mass and width induced by the hot medium. Figure 2 displays the ratio of the moment calculated from Eq. (3) using $\tilde{\alpha}(T)$ for G_2 . We can see that the ratio have a stable point from vacuum to $1.05T_c$ but the stability is no longer achieved beyond $1.06T_c$. Also as seen from the figure, the stable point shifts to larger n as temperature increases. However, Eq. (3) becomes worse as

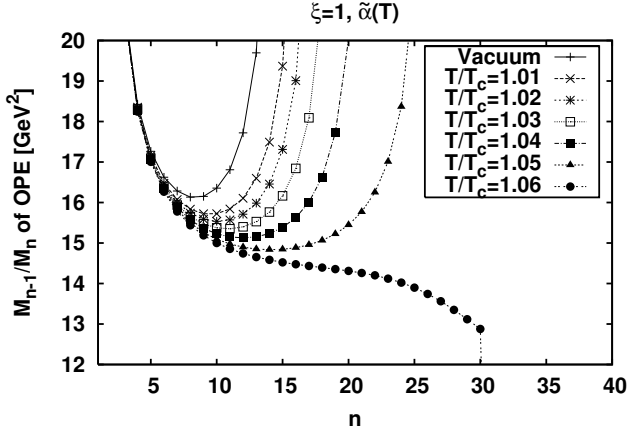


FIG. 2: Ratio of the moment from the OPE for various T/T_c .

n increases, because the Wilson coefficients increases with n [9]. This can be improved by increasing ξ , but the stability holds only up to $1.06T_c$ even for $\xi = 3.0$. If we use $\alpha_{qq}(T)$ for G_2 , the stability becomes worse due to its larger value at this temperature region. In the $\xi = 1$ case, there is no stable point for $T = 1.05T_c$ with $\alpha_{qq}(T)$. This lack of stability does not necessarily mean dissociation of J/ψ but shows a breakdown of our approximation. The reasons for the breakdown are twofold. One is the lack of convergence of the OPE in Eq. (3). This can be improved by including higher dimensional operators [24]. The other is physical one. Since the non-perturbative part largely decreases above T_c , perturbative contribution will become more important. Hence, in order to study higher temperature region, we will need to improve the phenomenological side to be more consistent with the OPE side, which can be accomplished by a temperature dependent continuum contribution. Then it will lead to n -independent results for physical parameters until the J/ψ really dissolves.

Before going to results of m and Γ , it is useful to see a feature of the phenomenological side. We depict the moment ratio of the phenomenological side based on the dispersion integral (4) in Fig. 3. We choose two n values which correspond to the stable points at vacuum and $T = 1.05T_c$, respectively. We see that the moment depends on Γ very weakly. For the larger n , the dependence becomes slightly stronger. On the other hand, the moment of the OPE side (Fig. 2) shows about 2 GeV^2 decrease from vacuum to $1.05T_c$. Hence, the width must become very large to achieve 2 GeV^2 reduction of the moment ratio if change of the mass is small. The process to determine the mass and the width is nothing but evaluating the intersection between the stable points in Fig. 2 and the curves in Fig. 3. Because of the monotonic behavior of the r_n as a function of m and Γ , mass satisfying the sum rule takes its minimum value in the $\Gamma \rightarrow 0$ limit, *i.e.*, magnitude of the mass shift becomes the largest if width stays constant. In this limit, mass is given by a simple relation $m^2 = r_n|_{\text{OPE}} - 4m_c^2\xi$ [9]. We cannot determine both mass and width only with the sum rule because it provides only one equation with respect to

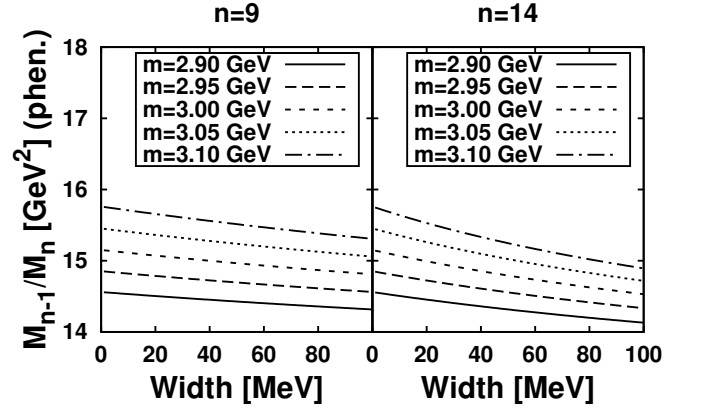


FIG. 3: The moment ratio of the phenomenological side as a function of Γ for various masses. Left figure shows $n = 9$ case and right one shows $n = 14$ case.

two unknown quantities, m and Γ . The situation is similar to light vector meson[25]. However, we can extract a relation between the mass shift and width by fixing the mass firstly and then solving $r_n|_{\text{OPE}} = r_n|_{\text{phen.}}$ for Γ because the monotonic behavior of the r_n guarantees the unique solution.

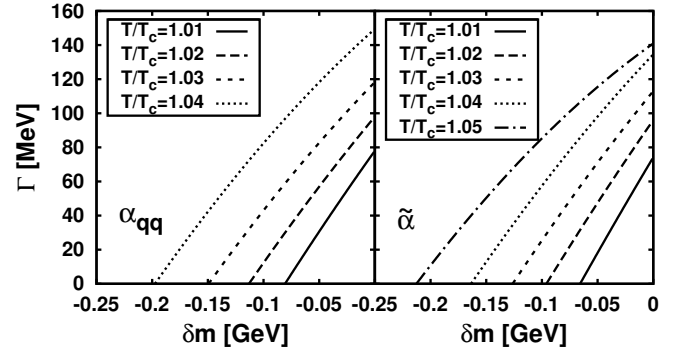


FIG. 4: Relation between mass shift and width at finite temperature. Left figure shows the case of $\alpha_{qq}(T)$ and right one shows the case of $\tilde{\alpha}(T)$.

In Fig. 4, we display the result of relation between mass shift $\delta m = m_{\text{medium}} - m_{\text{vacuum}}$ and width which satisfy the sum rule. The result gives a clear, almost linear relation between Γ and δm . The difference in $\alpha_s(T)$ appears as 10-50 MeV difference of the mass shift at $\Gamma = 0$ and about 10 MeV difference of the width at $\delta m = 0$.

Finally we plot the two extreme cases, $m_{J/\psi}$ for $\Gamma \rightarrow 0$ and Γ for $\delta m \rightarrow 0$, as a function of temperature in Fig. 5. In both cases, the change is almost linear with temperature. We stress that, however, these results are *extreme* cases. Our results show there must be notable change of mass or width, or both of them. Hence, once either mass or width is estimated by other methods, one can obtain the other through the relation given in Fig. 4. For example, a pQCD calculation can give thermal width [26]. This does not show large (~ 100 MeV) thermal width near T_c . Then the large mass shift is expected.

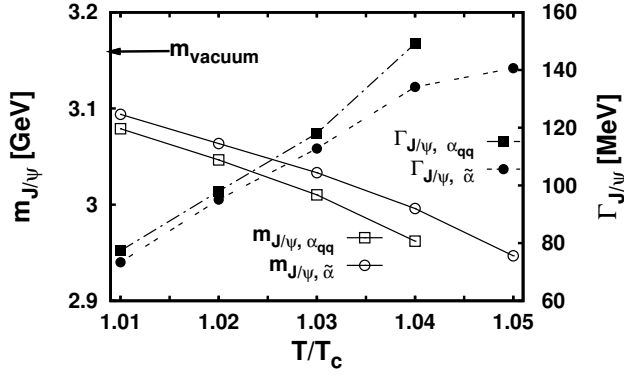


FIG. 5: Temperature dependence of the $m_{J/\psi}$ (left vertical axis) in $\Gamma \rightarrow 0$ limit (max. mass shift) and $\Gamma_{J/\psi}$ (right vertical axis) in $\delta m \rightarrow 0$ limit (no mass shift). Mass and width are indicated by open symbols and closed ones, respectively.

Such mass shift is in fact expected, as the sudden reduction of the asymptotic value of the potential just above T_c seen in a lattice QCD [27] will inevitably lead to lowering of bound state energy on that potential [28, 29]. Although the accuracy of the lattice MEM method is not enough for clear comparison, the spectral function calculated from a potential model motivated by a full-lattice QCD shows a large shift of the peak [30]. A recent full lattice QCD also shows the shift of the J/ψ peak [31].

In heavy ion experiments, this mass shift will also change the number of formed J/ψ according to statistical hadronization. For $T = 170$ MeV, mass decrease of 100 MeV increases the yield by a factor of 2 since the factor is roughly characterized by $e^{-\delta m/T}$. Furthermore, a hydrodynamic calculation shows the lifetime of QGP is $\sim 4 - 5$ fm/c at 200 GeV/A Au+Au collisions at RHIC [32]. This will be much longer at LHC. If the width broaden to as large as such lifetime of the plasma, the mass shift might be detectable.

In summary, we have given the first model-independent analysis of possible mass shift and broadening of width of J/ψ on the basis of lattice QCD inputs and QCD sum rule in the quenched approximation. Although the formalism, OPE up to dimension 4, is found to be applicable only to $T \simeq 1.05T_c$, we found that the change of gluon condensates in the deconfined phase causes notable reduction of the ratio of the moment $r_n|_{\text{OPE}}$, which results in mass shift and width broadening. The mass is found to be almost linear decrease with temperature if the width remains unchanged while the width linearly increases with temperature in the case of no mass shift. The maximum values are $\delta m \simeq 200$ MeV and $\Gamma \simeq 140$ MeV at $T = 1.05T_c$. Further details including results for different ξ will be given in a future publication [23].

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* Electronic address: morita@phy.yonsei.ac.kr

† Electronic address: suhoun@phy.yonsei.ac.kr

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